Theoretical and experimental comments on rf current drive phenomenon

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Radio-frequency current drive phenomenon in tokamak is reviewed both theoretically and experimentally. The quasi-linear theory often used is checked and problems are pointed out on the experimental point of view.   The plausable theoretical understanding is reviwed on the standpoint of the basic elecromagnetics and is compared to experimental results.   The current drive using cyclotoron wave is considered.

1. Introduction

The idea of passing current in torus is historically for the improvement of plasma confinement.   When the stellarator is struggling with poor confinement the tokamak machine was invented by Soviet union physicist of Tam and Sakharov and the plasma current flows by electromagnetic induction method to realize the better confinement rather than stellarator.   However, since the electromagnetic induction method to flow the plasma current I_p must become pulse operation of fusion reactor, the non-iductive current drive method is desired by the radio-frequency and the beam method. Now, the research into non-inductive current drives was first sparked by N. J. Fisch formulation using the quasi-linear theory in 1978[1].   In 1980, T. Yamamoto experimentally verified this by using a lower hybrid wave transformer showing the reduction of the loop voltage[2].   In the lower hybrid current drive (LHCD) the energy confinement time τ_E is improved and in JT-60 simultaneous injection of LHCD and neutral beam injection (NBI) heating τ_E is not degraded for increase of the heating power P_{in} against to the Kaye-Goldstone scaling ($\tau_E \propto P_{in}^{-0.58}$) prediction. Continuous opeation of tokamak is realized in the LHCD experiment of TRIAM-1 in passing I_P as long as 70 seconds.   Non-inductive current drive, especially radio-frequency current drive, promises to streamline furnace design of fusion reactor. However, progress in non-inductive current drive has recently stalled, and with lower hybrid waves, for which the most data has been accumulated, there is a problem in that the current values obtained experimentally are too large compared to the theory that is considered to be correct and are too small in the case of electron cyclotron current drive (ECCD).   In the next generation device, based on the predictions of quasi-linear theory that LHCD will only flow on the surface, it is designed that the current drive in the central region will be performed by negative ion beams or electron cyclotron current drive (ECCD), so the current drive is designed to by a NBI current drive(CD) and ECCD.   However, such a design is worth consideration.   In this paper, we focus on the quasi-linear theory that has been applied so far, clarify the validity of this theory and the limitations of application according experimnral results. The new plaubable theory of current drive phenomenon is clarified and the question on the cyclotorn current drive is pointed out.

2. Validity of the quasi-linear theory

In the quasi-linear theory understanding of the current flowing is formed by the asymmetry of distribution function between the positive and negative direction of travelling wave, whose positive di1rection one having the plateau is[1]

$$f_{quasi} = c\exp(-\int_{\square}^{v} \frac{2v/v_{the}^{\square}}{1+D_W/D_{coll}} dv),$$ (1)

where v_{the} is electron thermai velocity, D_w is quasi-linear diffusion coefficient and D_{coll} is the diffusion coefficient due to collision.   So the current expressed as

$$J = -e\int_{-\infty}^{\infty} v f_{quasi} dv.$$ (2)

The experimental verification of the quas-linear theory was perfomed by Robertson, where in the linear machine the electron beam is generated, making a bump of tail of df/dv > 0 in th electron velocity distribuntion function in this region generated instability spatial growth rate saturated with increase of beam current and the power spectrum of saturated satisfy the quasi-linear theory.   The problem of in the current drive case when the state of df/dv>0 is not satisfiyed. In the Fisch theory the current is formulated by the axisymmetric causes the current and the current is formed by the turbulent, however, the many ion coherent waves are in the turbulent state and the three dimensional k system must have infinitive energy is required and does not form the plateau in the distribution function.   On the experimental stand poit of view radio-frequency current drive in tokamak is often explained by the quasi-linear theory[1], however, many points of disagreement are pointed out in experiments.   First, the phase velocity of the injected rf travelling wave for the current flowing is much larger than the plateau regime on the electron distribution function, which indicates the current flowing is less smaller.   This is called the spectrum gap problem.   Second, much current must be flowing when the rf wave spectrum is much broader because of causing the more wide plateau resume in the distribution function.   Experimentally much current is flowing only for the sharp rf spectrum. Denoting eq. (2) to be

$$J_{rf} = en_{res}v_{eff},$$ (3)

Fig. 1 Plateau distribution function f(v_z) and rf power spectrum $P_{rf}(v_z)$
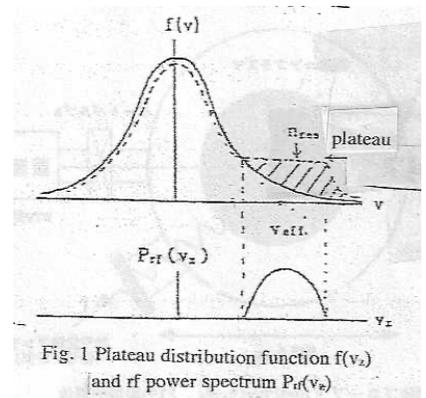
whete n_{res} is the number of resonant particles for causing the current and v_{eff} is the average velocity of the current flow.   In in Fisch theory n_{res} is determined by collision computing the Fokker Planck equation, however, v_{eff} is determined by the rf pectrium only.   In the induced spectrum, the larger phase velocity, the laeger v_{eff} and n_{res} is less smaller and the induced current becomes smaller.   This is the cause of spectrum gap.   In the quasi-linear undertstanding, the current is in maximum when the distribution function is stational.   In theory, the current reaches a maximum when the distribution function reaches a steady state, and the greater the phase velocity and the broader spectron causes much plaeau in the distribution function causing the current as shown in Fig. 1.

3. New theory of currnt drive by Stix and Kojima method [6]
     Stix has derived the average force per particle due to a traveling longitudinal electric field[5].   The basic understanding of the current flowing is the equation of motion applying the electric field E : $mdv/dt = eE - mnv$, where v is the collision frequency.

In the stational state(d/dt=0), $v = eE/m\nu$, the flowing current is $J = env = e^2n/m\nu E = \sigma E$, where $\sigma(=e^2n/m\nu)$ is the electric conductivity.   Plausible derivation of rf current is j $=\sigma E$ multiplying the didtribution fanction as $j_{rf} = \int_{-\infty}^{\infty} \sigma E_{eff} f dv$ [6]. Hence E_{eff} is the effective electric field receiving from the rf travelling wave.   The equation of motion of travelling wave electric field $E\cos(kz -\omega t)$ is

$$m\frac{dv}{dt} = e\ E\cos(kz - \omega t) \qquad (3)$$

and zeroth order motion is free streaming of RHS = 0, and, $v = v_0$ and $z = z_0$, $z = v_0 t - z_0$ when $t = 0$.   Subbstituting this equation in to eq. (1), considering Landau damping is initial value problem

$$v_1 = (eE/m\alpha)[\sin(kz_0 + \alpha t) - \sin kz_0]$$

$$z_1 = \int_0^t v_1 dt = \left(\frac{eE}{ma}\right)\left[-\frac{\cos(kz0+at)-\cos kz0}{a} - t\sin kz_0\right)]$$

where, $\alpha = kv_0 - \omega$.   Subsitututing $z = z_0 + v_0 t + z_1$ again into eq.(3) and we consider $kz_1 << 1$ and $\cos kz_1 = 1$, $\sin kz_1 = kz_1$ averaging on z_0, we obtain

$$<m\frac{dv}{dt}>z_0 = k\frac{e2E2}{2ma}\left(-\frac{\sin at}{\alpha^2} + \frac{t\cos at}{\alpha}\right) = k\frac{e2E2\pi}{2m}\delta'(a) \qquad (4)$$

$\delta'(a)$ means the derivative of delta function δ on α.   Eq.(4) express average force receiving from rf travelling wave and we can express $<mdv/dt>_{z0} = eE_{eff}$.   The electric conductivity σ is colloision frequency ν is expressed by $\nu = 1/\tau$ and using Spitzer conductivity $\tau = 2\pi Nv^3/\omega_{pe}^4 \ln\Lambda = \tau_v v^3/V^3$ is used, where, τ_v is t when the phase velocity is $V = \omega/k$ and ω_{pe} is the electron plasma frequency and $\ln\Lambda$ is Coulomb Logarithm and the current is expressed   by σE_{eff}, and integrate this equation multiplying the distribution function $f(v)$ as

$$J_{rf} = \int_{-\infty}^{\infty} \sigma E_{eff}\ f dv = \int_{-\infty}^{\infty}\frac{Ne^2}{m}\tau_v\frac{v^3}{V^3}k\frac{eE^2\pi}{2m}\delta'(\alpha)f(v)dv = -\frac{\pi Ne^3E^2\tau_v}{2m^2v^3}\left|\frac{\omega+\alpha}{k}\right|\delta'(\alpha)d\alpha$$

Performing partial integral this becomes

$$J_{rf} = \frac{e\omega_{pe}^2}{8mV^3}\int_{-\infty}^{\infty}\frac{d}{d\alpha}\left\{f(v)\left|(\frac{\omega+\alpha}{k})^3\right|\right\}\delta(\alpha)d\alpha$$

Finally, we get

$$J_{rf} = \frac{e\omega_{pe}^2E^2\tau_v}{8mk}\{f'(v) + 3\frac{f(v)}{V}\} \qquad (5)$$

4. Question current drive due to cyclotron damping

In the cyclotron current drive case it is noted that the rf wave electric field is perpendicular to the magnetic field denoting as $E_\perp$.   The equation of motion is

$$m \frac{v_\perp}{dt} = eE_\perp - \frac{m}{\tau} v_\perp$$

$$m \frac{dv_{//}}{dt} = - \frac{m}{\tau} v_{//}$$

In stationary state(d/dt = 0), we solve on $v_{//}$ assuming when t = 0 then $v_{//} = v_0$, then $v_/ = v_0 \exp(-t/\tau) = v_0 \exp(eE_\perp/mv_\perp)t$, so current J is

$$j = eNv_{//} = eNv_o \exp(-eE_\perp/mv_\perp)t. \qquad (6)$$

This equation means the parallel current caused by the perpendicular rf electric field $E_\perp$, where the resistivity is reduced by $E_\perp$ and the decay time $\tau(=mv_\perp/eE_\perp)$ is only enlarged and it is noted that the current flows by cyclotron wave is too small. In the flowing current cyclotron wave is on before the RF on time increased does not increaaase. That is, turning on t he ECH wave at a certain time will not result in the current exceeding the value before the rf was turned on as shown in Fig.2. Third, the current drive using the cyclotron wave can also flow due to the perpendicular heating, whose current must be saying 3/4 of the case of the LHCD case[4]. However, experimantally the current drive using cycloron wave is much smaller than that by LHCD, which is not three quarter of the LH wave, but is below one tenth of it[5]. It seems unlikely that the current would flow even three-quarters of that in the case of Landau damping. However, it is possible that a parallel rf electric field is generated during the propagation of the cyclotron wave and drive the current through Landau damping.

5. Conclusion

The main theme of current drive is improving the confinement and the first priority is that confinement is better than in case of no current whether it is inductive or non-inductive current drive. In that respect, LHCD has so far passed. The spectral gap in the quasi-linear theory that has been conventionally applied to rf current drive, and the essential discrepancy between the drive current value and the distribution of the rf spectrum, have been questioned from the perspective of basic principles of electromagnetism, and a fundamental understanding using Landau damping has been proposed. Questions have also been raised about the mechanism of current drive by cyclotron damping.

References
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