Numerical investigation of the O-X mode conversion in a tokamak plasma

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Introduction Electron cyclotron resonant heating (ECRH) is a commonly used technique for plasma heating and current drive in magnetic confinement fusion [1]. However, when it comes to overdensed plasma, such as the ones in spherical tokamaks, it can be limited due to density cut-offs [2]. An appealing alternative to ECRH is heating with electron Bernstein waves (EBWs), as they are purely electrostatic and are not limited by the density cut-offs. On the other hand, EBWs cannot propagate in vacuum, so their excitation is only possible inside the plasma [3]. One approach that has been investigated in the past decades is the so-called O-X-B mode conversion, where an O-mode wave is first converted into an X-mode, which in turn couples to the EBW. In this report, the first step of the process (O-X) is numerically investigated for a simplified slab geometry, as well as for a more realistic D-shaped plasma profile, with and without the addition of edge density perturbations.

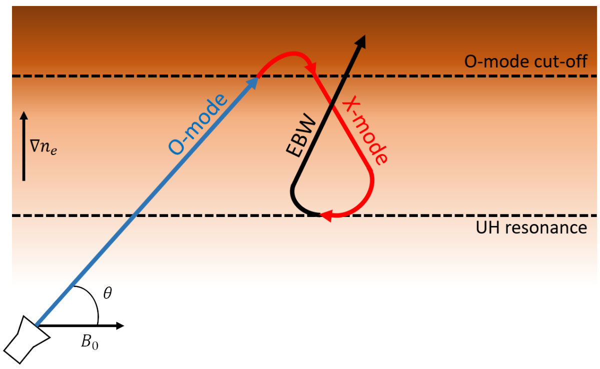

Figure 1: *O-X-B mode conversion process.*

The O-X-B mode conversion is a two-step process, as it is illustrated in Fig. 1. During the first step, an O-mode wave is injected into the torus under an oblique angle with respect to the background magnetic field. The wave is propagating inside the plasma until it reaches the O-mode cut-off density. At the optimum angle, the O-mode wave coincides with the X-mode wave, as they share the same phase and group velocity. The O-mode wave is then converted into a pure X-mode wave [3]. At the second step, the converted wave is propagating backwards until it reaches the upper hybrid resonance (UHR). At the UHR the X-mode wave is coupling to the EBWs. Finally, the EBWs are propagating inside the plasma until they are absorbed by the cyclotron interaction near the harmonic resonance.

FDTD code A 2D finite-difference time-domain (FDTD) code is used to investigate the O-X conversion efficiency. Based on Yee's algorithm [4], Maxwell's equations are solved on a Cartesian grid

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \tag{1}$$

$$\frac{\partial \mathbf{E}}{\partial t} = c^2 \nabla \times \mathbf{B} - \frac{1}{\varepsilon_0}\mathbf{J}, \tag{2}$$

where c is the speed of light in vacuum and ε_0 the vacuum permittivity. The plasma effects are captured by the current density $(\mathbf{J})$, which is calculated through the fluid equation of motion for the electrons, in phase with the electric field

$$\frac{\partial \mathbf{J}}{\partial t} = \varepsilon_0 \omega_{pe}^2 \mathbf{E} - \omega_{ce} \mathbf{J} \times \hat{\mathbf{B}}_0, \tag{3}$$

with $\omega_{pe} = \sqrt{n_e e^2/(\varepsilon_0 m_e)}$ the electron plasma frequency (e the elementary charge and m_e the electron mass), $\omega_{ce} = |e|B_0/m_e$ the electron cyclotron frequency and $\hat{\mathbf{B}}_0$ the unit vector into the direction of the background magnetic field.

Plasma slab The first scenario under investigation is a simple plasma slab, with a linear density gradient along the direction of the wave propagation. The normalized density scale length $k_0 L_n$ is used to control the slope of the density profile,

$$n_e(x) = \frac{2\pi}{k_0 L_n} x. \tag{4}$$

An 1D density profile is plotted in Fig. 2 (top). The angle of injection is varied in order to investigate the influence on the conversion efficiency. The result is shown in Fig. 2 (bottom) and it can be seen that it is in agreement with the theoretical result, in terms of the shape, with the peak conversion efficiency happening at $\theta \approx 50^\circ$. The absolute values are different due to the injected polarization mismatch. In the theoretical result, the injected wave is assumed to be a pure O-mode for any angle, while in the simulation domain the injected wave is a mixture between O- and X-mode.

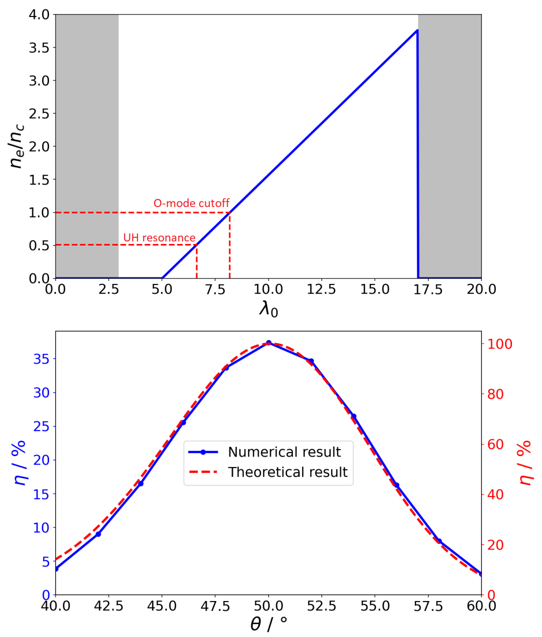

Figure 2: (Top) *Density profile of plasma slab and* (bottom) *conversion efficiency result against theoretical prediction.*

D-shaped plasma The next step is the investigation of a more realistic profile, like a D-shaped tokamak profile. In order to replicate the steep pedestal of an H-mode discharge, the following modified hyperbolic tangent function is used [5]:

$$\text{mtanh}(R,\vec{x}) = L(z)\left(h + \frac{awz}{4}\right) \tag{5}$$

$$z = -4\frac{R - R_p}{w} \tag{6}$$

$$L(y) = \frac{1}{1 + e^{-y}} \tag{7}$$

where R_p is the pedestal radial location, h the pedestal height, w the pedestal width and a the profile gradient beyond the pedestal top. An example of such a profile is shown in Fig. 3 (top). The slab profile is also shown for reference.

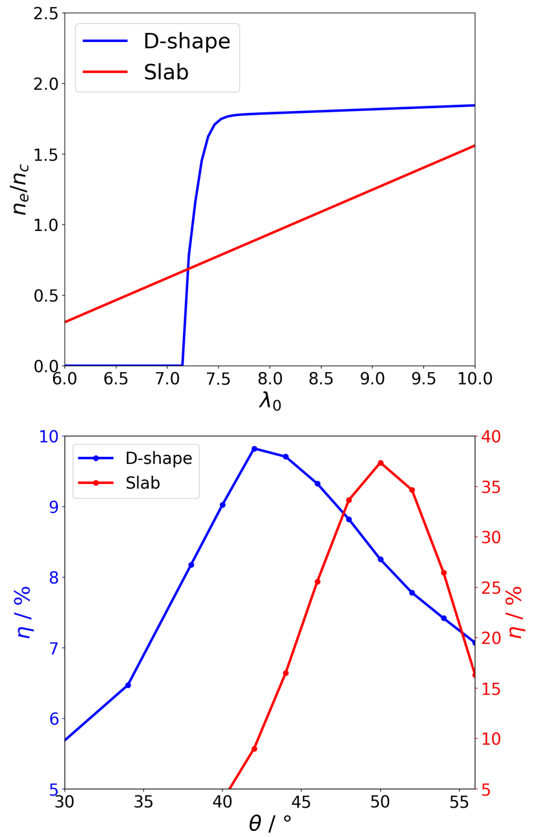

Figure 3: (Top) *Radial density profile of D-shaped plasma* (bottom) *conversion efficiency result against slab scenario.*

The conversion efficiency is plotted in Fig. 3 (bottom). Interestingly, the peak of the conversion is shifted to $\theta \approx 42^\circ$, while the overall conversion is lower by a factor of $\sim$3.5. The distribution appears to be wider, which is caused by the steeper plasma profile. Examples of the wave electric field for $\theta = 90^\circ$ and $\theta = 42^\circ$ are shown in Fig. 4. Note that for $\theta = 42^\circ$, part of the wave is not reflected, but trapped inside the pedestal region of the plasma, which demonstrates the successful O-X conversion as the X-mode wave is not able to pass the UHR.

Effect of edge density perturbations The final scenario under investigation is the D-shaped profile with an added density perturbation at the pedestal region. A Gaussian perturbation is added to the background density profile:

$$\tilde{n}_e = A\cos(m\theta)\exp\left(-\left(\frac{\rho - \rho_0}{\sqrt{2}\sigma_{ELM}}\right)^2\right) \tag{8}$$

where A is the peak density of the perturbation, m the poloidal mode number and σ_{ELM} the width. For this particular case the parameters $A = 0.8, m = 8$ and $\sigma_{ELM} = 0.08$ were selected. The conversion efficiency result is shown in Fig. 5. The peak remains at $\theta = 42^\circ$, but the overall conversion is increasing for the whole range of injection angles. This could be explained by a local mode conversion.

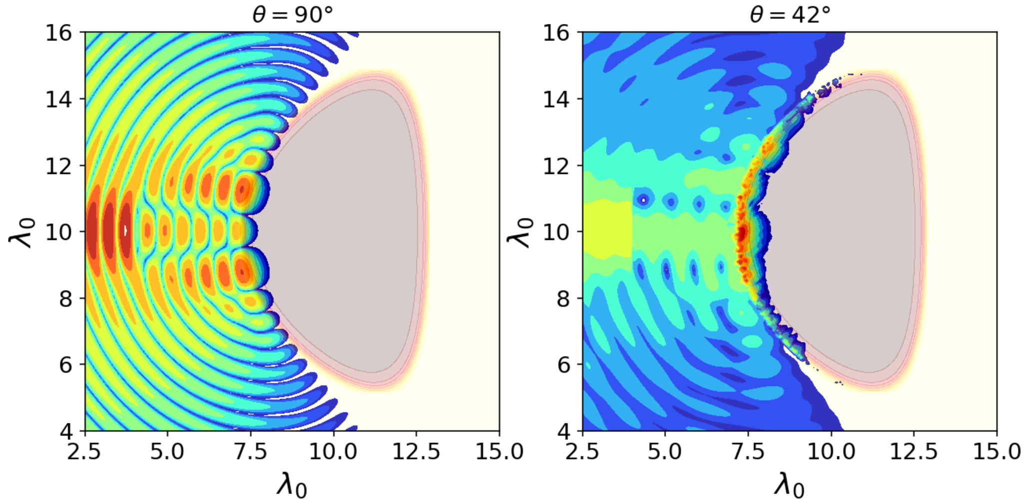

Figure 4: *Wave propagation for* $\theta = 90°$ (Left) *and* $\theta = 42°$ (Right).

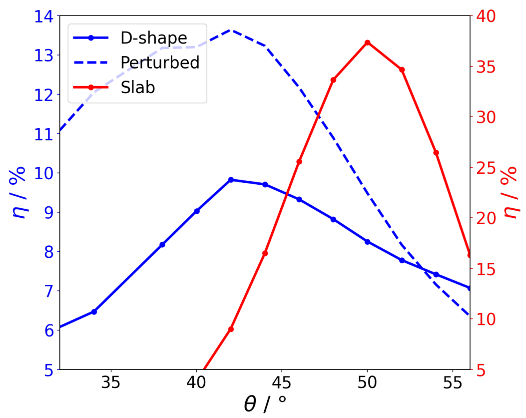

Figure 5: *Conversion efficiency for the perturbed D-shaped plasma.*

Conclusions An FDTD code has been applied to simulate the O-X mode conversion in different scenarios. Starting from a simple plasma slab, the theoretical result was confirmed. When moving to a more realistic D-shaped profile, the conversion efficiency was found to be significantly reduced, while the distribution became wider, which is attributed to the steeper gradient. The added perturbation on the D-shaped profile led to an increase of the conversion efficiency. This is thought to be due to local mode conversion effect. The next step of this project is to fix polarization of the injected wave, in order to introduce a pure O-mode wave for every injection angle.

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